ERRATA

2-DIMENSIONAL CATEGORIES
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NILES JOHNSON AND DONALD YAU

CONTENTS

This document contains known errors and corrections as of 01 February 2022. If you are aware of further errors, please contact both authors by email:

johnson.5320@osu.edu, yau.22@osu.edu.

The most recent version of these errata will be available on the authors' websites.

Page numbers refer to the published version of the text.

1. PAGE 2, PROPOSITION 1.1.3 (a)

The third sentence of this proof, "Since $\{x, y\} \in \mathcal{P}(x \cup y) \ldots$", should be replaced with the following.

> By Axiom (2) twice, $\mathcal{P}(\mathcal{P}(x \cup y)) \in \mathcal{U}$. Since $\{x, y\} \in \mathcal{P}(\mathcal{P}(x \cup y))$, Axiom (1) implies $\{x, y\} \in \mathcal{U}$.

2. PAGE 243, EXPLANATION 6.1.3, ITEM (3)

In the bottom right triangle, the slanted arrow should be 1_g, not 1_f.

3. PAGE 320, LEMMA 8.3.12

In this proof, each instance of the left unitor ℓ should be replaced with the right unitor r because its components

$$r_f : f1_A \xrightarrow{\cong} f \quad \text{and} \quad r_{f'} : f'1_A \xrightarrow{\cong} f'$$

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provide the desired natural isomorphism. Moreover, the first and third displays should use single arrows $\longrightarrow$ instead of double arrows $\Rightarrow$ as that is our convention outside of cell diagrams. The specific changes are as follows.

3.1. Sentence 3 should be replaced with the following.

> Thus the right unitor defines a natural isomorphism
> $$r : e_A \circ \mathcal{Y} \longrightarrow \mathrm{Id}_{\mathsf{B}(A,B)}.$$

3.2. Sentence 5: replace "the left unitor ℓ" with "the right unitor r".

3.3. Display 3 should be replaced with the following.

> $$\mathcal{Y} \circ e_A \longrightarrow \mathrm{Id}_{\mathsf{Str}(\mathcal{Y}_A, \mathcal{Y}_B)}.$$

4. Page 320, Corollary 8.3.13

This result and the paragraph immediately before it are incorrect as stated. They should be replaced by the following corrected version.

> Examining the proof of Lemma 8.3.12, we note that the modification components constructed there will be identities if B has trivial unitors and each θ is a strict transformation. Recall from Corollary 8.2.16 that $\mathcal{Y}$ is a 2-functor if B is a 2-category. Moreover, in that case then each $\mathcal{Y}_f$ is a strict transformation between 2-functors. Thus, we have the following.
>
> **Corollary 8.3.13.** *If B is a 2-category, then for each pair of objects A and B in B, the Yoneda 2-functor $\mathcal{Y}$ provides an isomorphism of categories*
> $$\mathcal{Y} : \mathsf{B}(A, B) \xrightarrow{\cong} \mathsf{Strict}(\mathcal{Y}_A, \mathcal{Y}_B)$$
> *where*
> $$\mathsf{Strict}(\mathcal{Y}_A, \mathcal{Y}_B) = \mathsf{2Cat}(\mathsf{B}^{\mathsf{op}}, \mathsf{Cat})(\mathcal{Y}_A, \mathcal{Y}_B)$$
> *denotes the category of strict transformations and modifications between $\mathcal{Y}_A$ and $\mathcal{Y}_B$.*

The correction to Corollary 8.3.13 also requires corrections to Corollary 8.4.2 on page 328, noted below.

5. Page 328, Corollary 8.4.2

Make the following changes in the paragraph before Corollary 8.4.2.

5.1. In the first sentence, replace "Corollary 8.3.13" with "Corollary 8.2.16".

5.2. The second sentence should be changed to reflect the correction to Corollary 8.3.13 above. It should be replaced by the following two sentences.

> If B is a 2-category, then Corollary 8.3.13 shows that $\mathcal{Y}$ induces a local isomorphism to $\mathsf{2Cat}(\mathsf{B}^{\mathsf{op}}, \mathsf{Cat})$, the 2-category consisting of 2-functors from B^{op} to Cat together with strict transformations and modifications. Therefore, by the 2-categorical Whitehead Theorem 7.5.8, we obtain the following.

5.3. Corollary 8.4.2 is incorrect as stated. It should be replaced with the following corrected version.

> **Corollary 8.4.2.** *If* B *is a 2-category, let* stB_2 *denote the essential image of*
> $$\mathcal{Y} : \mathsf{B} \longrightarrow \mathsf{2Cat}(\mathsf{B}^{\mathsf{op}}, \mathsf{Cat}).$$
> *Then*
> $$\mathsf{B} \longrightarrow \mathsf{stB}_2$$
> *is a 2-equivalence.*

6. Page 402, Definition 10.4.3

Add the following sentence to the definition of $F^2_{f,g}$, just after the sentence containing (10.4.6).

> Naturality of $F^2_{f,g}$ with respect to morphisms $e : Z \longrightarrow Z'$ in C is verified in the proof of Lemma 10.4.7.

7. Page 403, Lemma 10.4.7

The proof of Lemma 10.4.7 should include an explanation of the naturality of $(F^2_{f,g})_Z$ with respect to morphisms in C. The following paragraph should be added after line 5, just before the sentence about naturality of F^2 in the sense of (4.1.6).

To show that each $F^2_{f,g}$ is a 2-cell in Cat, for composable morphisms

$$A \xrightarrow{f} B \xrightarrow{g} C \quad \text{in} \quad \mathsf{C},$$

we show that the components $(F^2_{f,g})_Z$ are natural with respect to morphisms

$$e : Z \longrightarrow Z' \quad \text{in} \quad P^{-1}(C).$$

The desired naturality square is the boundary of the following diagram in $P^{-1}(A)$.

$$\begin{tikzcd}
Z_{g,f} \arrow[rr, "(F^2_{f,g})_Z"] \arrow[dr, "\overline{g}\overline{f}"'] \arrow[ddd, "(f^F g^F)e"'] & & Z_{gf} \arrow[dl, "\overline{gf}"] \arrow[ddd, "(gf)^F e"] \\
& Z \arrow[d, "e"] & \\
& Z' & \\
Z'_{g,f} \arrow[rr, "(F^2_{f,g})_{Z'}"'] \arrow[ur, "\overline{g}'\overline{f}'"] & & Z'_{gf} \arrow[ul, "\overline{gf}'"']
\end{tikzcd}$$

In the above diagram, the two triangles commute by (10.4.6) for Z and Z', respectively. The left trapezoid commutes by (10.4.5) for $(g^F e, \overline{f}, \overline{f}')$ and $(e, \overline{g}, \overline{g}')$. The right trapezoid commutes by (10.4.5) for $(e, \overline{gf}, \overline{gf}')$. Commutativity of the inner regions and invertibility of $(F^2_{f,g})_Z$ implies that the composite

$$Z_{gf} \xrightarrow{(F^2_{f,g})^{-1}_Z} Z_{g,f} \xrightarrow{(f^F g^F)e} Z'_{g,f} \xrightarrow{(F^2_{g,f})_{Z'}} Z'_{gf}$$

along the top, left, and bottom is a raise for $\langle \overline{gf}', e(\overline{gf}); 1_A\rangle$. Therefore the composite above is equal to $(gf)^F e$ by the uniqueness of raises for Cartesian morphisms as in (10.4.5).